CHAPTER 22: MATERIALS SELECTION
ECONOMIC, ENVIRON., & DESIGN ISSUES

ISSUES TO ADDRESS...

- **Price and availability of materials.**
- **How do we select materials based on optimal performance?**
- **Applications:**
  --shafts under torsion
  --bars under tension
  --plates under bending
  --materials for a magnetic coil.

PRICE AND AVAILABILITY

- **Current Prices on the web[a]:**

  --Short term trends: fluctuations due to supply/demand.

  --Long term trend: prices will increase as rich deposits are depleted.

- **Materials require energy to process them:**

--Energy to produce materials (GJ/ton)

Al	237 (17)[b]
PET	103 (13)[c]
Cu	97 (20)[b]
steel	20[d]
glass	13[e]
paper	9[f]

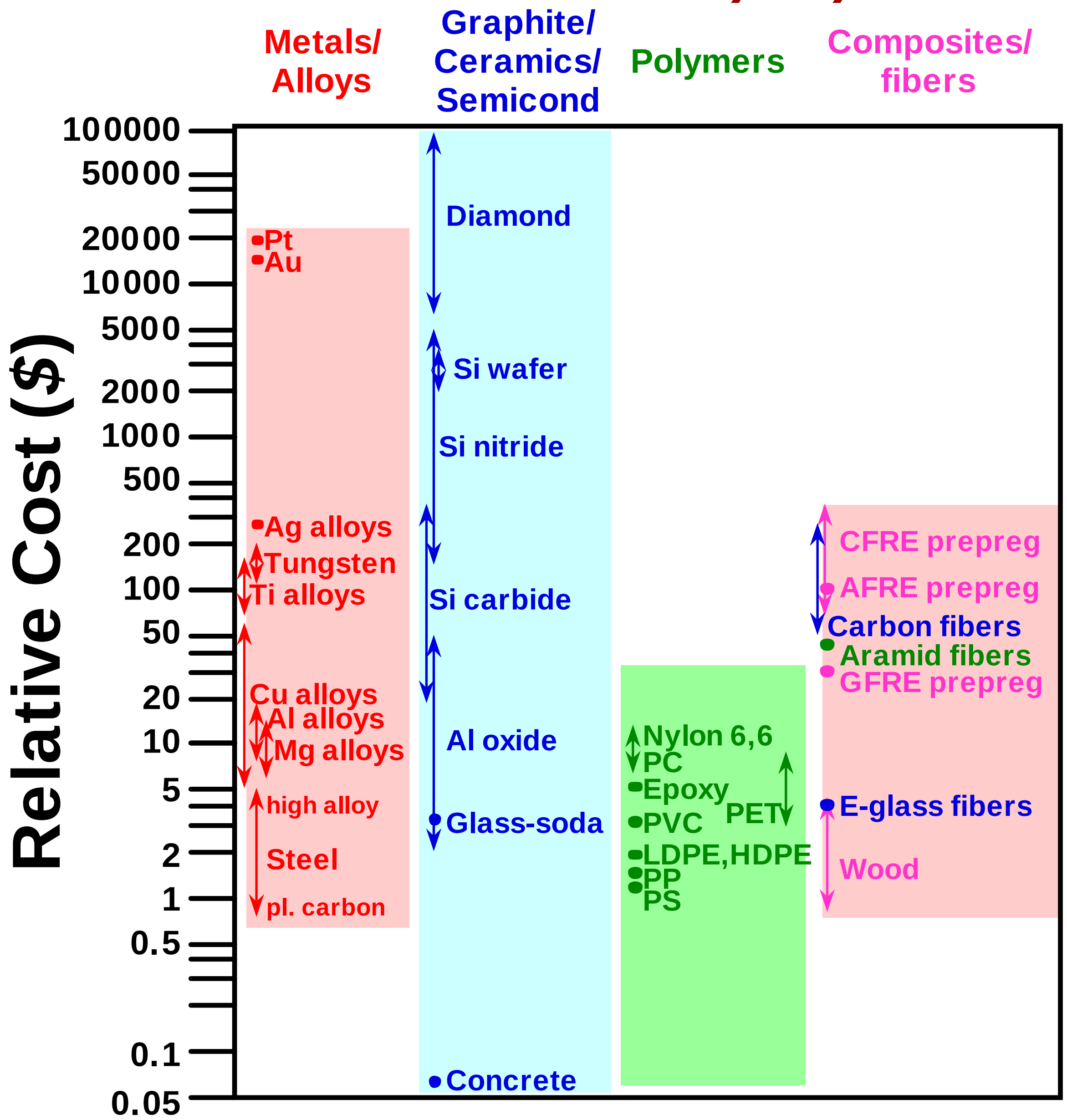
Energy using recycled material indicated in green.

--Cost of energy used in processing materials ($/GJ)[g]

elect resistance	25
propane	11
natural gas	9
oil	8

a http://www.statcan.ca/english/pgdb/economy/primary/prim44.htm
a http://www.metalprices.com
b http://www.automotive.copper.org/recyclability.htm
c http://members.aol.com/profchm/escalant.html
d http://www.steel.org.facts/power/energy.htm
e http://eren.doe.gov/EE/industry_glass.html
f http://www.aifq.qc.ca/english/industry/energy.html#1
g http://www.wren.doe.gov/consumerinfo/rebriefs/cb5.html

RELATIVE COST, $, OF MATERIALS

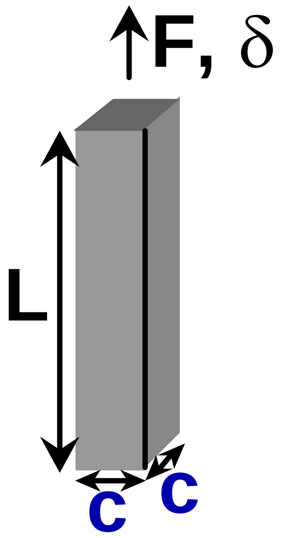


$$\$ = \frac{\$/\text{kg}}{(\$/\text{kg})_{\text{ref material}}}$$

- **Reference material:**
  --Rolled A36 plain carbon steel.
- **Relative cost, $, fluctuates less over time than actual cost.**

Based on data in Appendix C, *Callister, 6e*.
AFRE, GFRE, & CFRE = Aramid, Glass, & Carbon fiber reinforced epoxy composites.

STIFF & LIGHT TENSION MEMBERS



- **Bar must not lengthen by more than δ under force F; must have initial length L.**

-- Stiffness relation:

$$\frac{F}{c^2} = E\frac{\delta}{L} \qquad (\sigma = E\varepsilon)$$

-- Mass of bar:

$$M = \rho L c^2$$

- **Eliminate the "free" design parameter, c:**

$$M = \frac{FL^2}{\delta}\frac{\rho}{E}$$

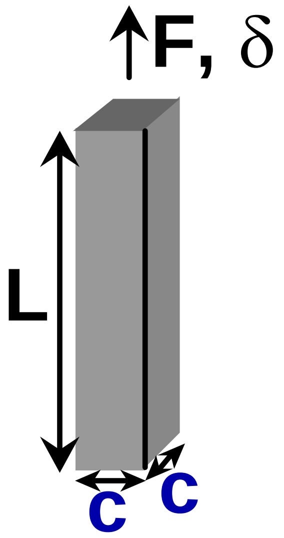
specified by application

minimize for small M

- **Maximize the Performance Index:**
(stiff, light tension members)

$$P = \frac{E}{\rho}$$

STRONG & LIGHT TENSION MEMBERS



- Bar must carry a force F without failing; must have initial length L.

  -- Strength relation:

  $$\frac{\sigma_f}{N} = \frac{F}{c^2}$$

  -- Mass of bar:

  $$M = \rho L c^2$$

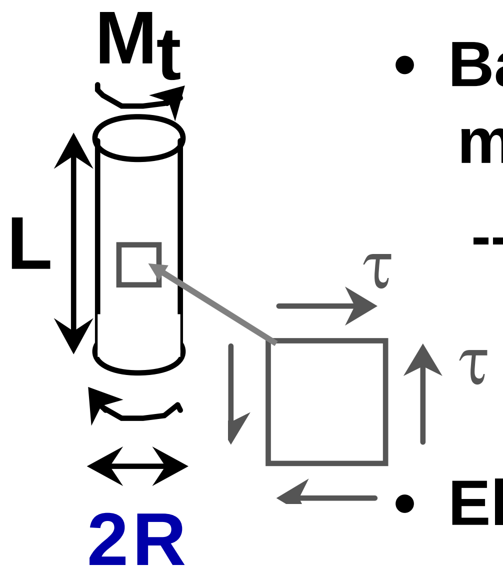
- Eliminate the "free" design parameter, c:

$$M = \underbrace{FLN}_{\text{specified by application}} \underbrace{\frac{\rho}{\sigma_f}}_{\text{minimize for small M}}$$

- Maximize the Performance Index: $P = \dfrac{\sigma_f}{\rho}$

  (strong, light tension members)

STRONG & LIGHT TORSION MEMBERS



- **Bar must carry a moment, M_t ; must have a length L.**

-- Strength relation:

$$\frac{\tau_f}{N} = \frac{2M_t}{\pi R^3}$$

-- Mass of bar:

$$M = \rho \pi R^2 L$$

- **Eliminate the "free" design parameter, R:**

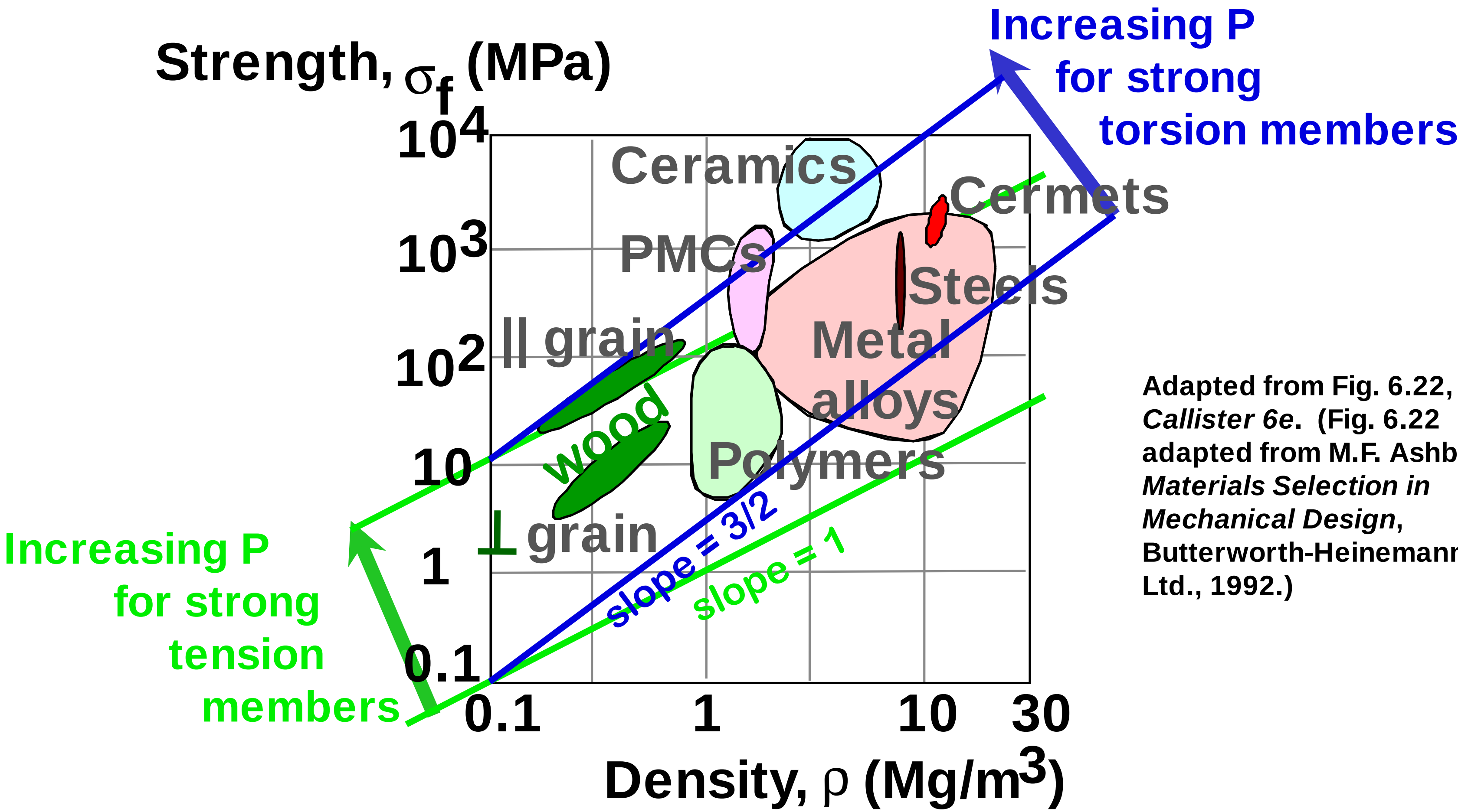
$$M = \left(2\sqrt{\pi}\, N M_t\right)^{2/3} L \, \frac{\rho}{\tau_f^{2/3}}$$

specified by application **minimize for small M**

- **Maximize the Performance Index:** **(strong, light torsion members)**

$$P = \frac{\tau_f^{2/3}}{\rho}$$

DATA: STRONG & LIGHT TENSION/TORSION MEMBERS

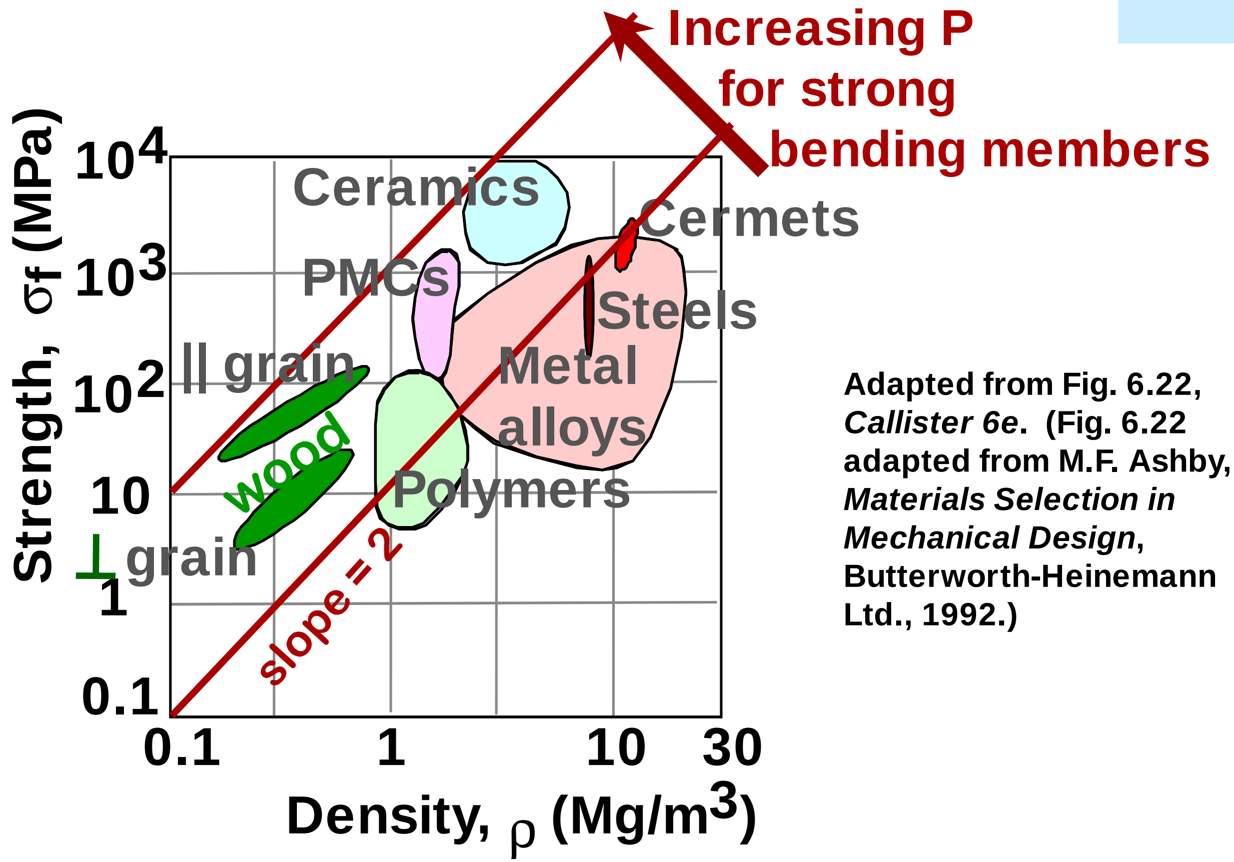


Adapted from Fig. 6.22, *Callister 6e*. (Fig. 6.22 adapted from M.F. Ashby, *Materials Selection in Mechanical Design*, Butterworth-Heinemann Ltd., 1992.)

DATA: STRONG & LIGHT BENDING MEMBERS

- **Maximize the Performance Index:**

$$P = \frac{\sigma^{1/2}}{\rho}$$



Adapted from Fig. 6.22, *Callister 6e*. (Fig. 6.22 adapted from M.F. Ashby, *Materials Selection in Mechanical Design*, Butterworth-Heinemann Ltd., 1992.)

DETAILED STUDY I: STRONG, LIGHT TORSION MEMBERS

- **Maximize the Performance Index:** $$P = \frac{\tau_f^{2/3}}{\rho}$$
- **Other factors:**
  --require σ_f > 300MPa.
  --Rule out ceramics and glasses: K_{Ic} too small.
- **Numerical Data:**

material	ρ (Mg/m^3)	τ_f (MPa)	P (MPa)$^{2/3}$m^3/Mg)
CFRE (v_f=0.65)	1.5	1140	73
GFRE (v_f=0.65)	2.0	1060	52
Al alloy (2024-T6)	2.8	300	16
Ti alloy (Ti-6Al-4V)	4.4	525	15
4340 steel (oil quench & temper)	7.8	780	11

Data from Table 6.6, *Callister 6e*.

- **Lightest: Carbon fiber reinf. epoxy (CFRE) member.**

DETAILED STUDY I: STRONG, LOW COST TORSION MEMBERS

- **Minimize Cost: Cost Index ~ M$ ~ $/P (since M ~ 1/P)**
- **Numerical Data:**

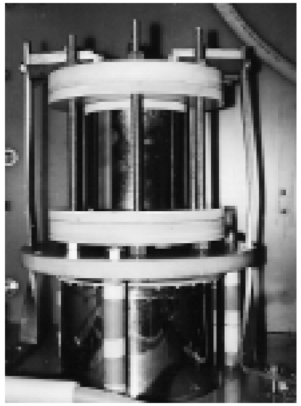
material	P (MPa)$^{2/3}$m^3/Mg)	*$*	($/P)x100
CFRE (vf=0.65)	73	80	112
GFRE (vf=0.65)	52	40	76
Al alloy (2024-T6)	16	15	93
Ti alloy (Ti-6Al-4V)	15	110	748
4340 steel (oil quench & temper)	11	5	46

Data from Table 6.7, *Callister 6e*.

- **Lowest cost: 4340 steel (oil quench & temper)**
- **Need to consider machining, joining costs also.**

DETAILED STUDY II: OPTIMAL MAGNET COIL MATERIAL

- **Background[2]: High magnetic fields permit study of:**
  - --electron energy levels,
  - --conditions for superconductivity
  - --conversion of insulators into conductors.
- **Largest Example:**
  - --short pulse of 800,000 gauss
    (Earth's magnetic field: ~ 0.5 Gauss)
- **Technical Challenges*:***
  - --Intense resistive heating can melt the coil.
  - --Lorentz stress can exceed the material strength.
- **Goal: Select an optimal coil material.**



Pulsed magnetic capable of 600,000 gauss field during 20ms period.

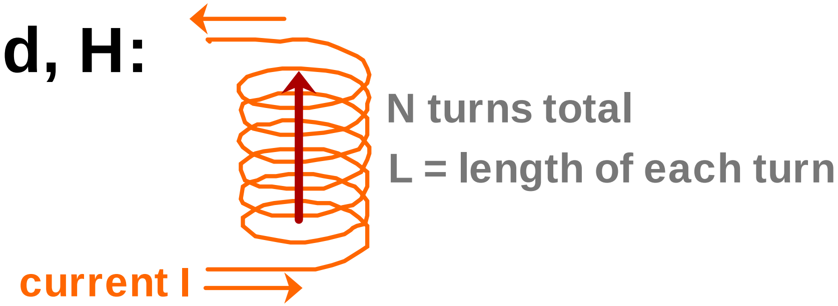
Fractured magnet coil.
(Photos taken at NHMFL, Los Alamos National Labs, NM (Apr. 2002) by P.M. Anderson)

(1) Based on discussions with Greg Boebinger, Dwight Rickel, and James Sims, National High Magnetic Field Lab (NHMFL), Los Alamos National Labs, NM (April, 2002).
(2) See G. Boebinger, Al Passner, and Joze Bevk, "Building World Record Magnets", Scientific American, pp. 58-66, June 1995, for more information.

LORENTZ STRESS & HEATING

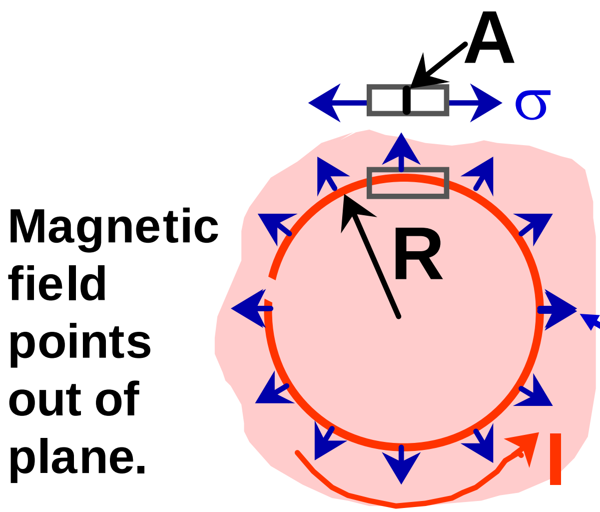
- **Applied magnetic field, H:**

H = N I/L



- **Lorentz "hoop" stress:**

$$\sigma = \frac{I\mu_o HR}{A} \quad \left(\leq \frac{\sigma_f}{N}\right)$$



$$\frac{\text{Force}}{\text{length}} = I\mu_o H$$

- **Resistive heating:**

(adiabatic)

$$\Delta T = \frac{I^2 \rho_e}{A^2 c_v}\Delta t \quad (< \Delta T_{max})$$

elect. resistivity

temp increase during current pulse of Δt

specific heat

MAGNET COIL: PERFORMANCE INDEX

- **Mass of coil:**

$$M = \rho_d A L$$

- **Applied magnetic field:**

$$H = N\, I/L$$

- **Eliminate "free" design parameters A, I from the stress & heating equations (previous slide):**

--Stress requirement

$$\frac{H^2}{M} \leq \frac{1}{2\pi R^2 L \mu_o N} \frac{\sigma_f}{\rho_d}$$

--Heating requirement

$$\frac{H\sqrt{\Delta t}}{M} \leq \frac{\sqrt{\Delta T_{max}}}{\sqrt{2\pi}\, RL} \frac{1}{\rho_d} \sqrt{\frac{c_v}{\rho_e}}$$

specified by application

Performance Index P_1: maximize for large H^2/M

specified by application

Performance Index P_2: maximize for large $Ht^{1/2}/M$

MAGNET COIL: COST INDEX

- **Relative cost of coil:**

  $\$ = \$_M$

- **Applied magnetic field:**

  $H = NI/L$

- **Eliminate M from the stress & heating equations:**

--Stress requirement

$$\frac{H^2}{\$} \leq \frac{1}{2\pi R^2 L \mu_o N} \frac{\sigma_f}{\rho_d \$}$$

specified by application

Cost Index C_1: maximize for large $H^2/\$$

--Heating requirement

$$\frac{H\sqrt{\Delta t}}{\$} \leq \frac{\sqrt{\Delta T_{max}}}{\sqrt{2\pi RL}} \frac{1}{\rho_d \$} \sqrt{\frac{c_v}{\rho_e}}$$

specified by application

Cost Index C_2: maximize for large $Ht^{1/2}/\$$

INDICES FOR A COIL MATERIAL

- **Data from Appendices B and C, *Callister 6e*:**

Material	σ_f	ρ_d	$	c_v	ρ_e	P_1	P_2	C_1	C_2
1020 steel (an)	395	7.85	0.8	486	1.60	50	2	63	2.5
1100 Al (an)	90	2.71	12.3	904	0.29	33	21	3	1.7
7075 Al (T6)	572	2.80	13.4	960	0.52	204	15	15	1.1
11000 Cu (an)	220	8.89	7.9	385	0.17	25	5	3	0.6
17200 Be-Cu (st)	475	8.25	51.4	420	0.57	58	3	1	<0.1
71500 Cu-Ni (hr)	380	8.94	12.9	380	3.75	43	1	3	<0.1
Pt	145	21.5	1.8e4	132	1.06	7	19	<1	<0.1
Ag (an)	170	10.5	271	235	0.15	16	<1	<1	<0.1
Ni 200	462	8.89	31.4	456	0.95	52	2	2	<0.1
units	MPa	g/cm^3	--	J/kg-K	$\Omega\text{-}m^3$	σ_f/ρ_d	$\frac{(c_v/\rho_e)^{0.5}}{\rho_d}$	$P_1/\$$	$P_2/\$$

Avg. values used. an = annealed; T6 = heat treated & aged;
st = solution heat treated; hr = hot rolled

- Lightest for a given H: 7075 Al (T6) ← P_1
- Lightest for a given $H(\Delta t)^{0.5}$: 1100 Al (an) ← P_2
- Lowest cost for a given H: 1020 steel (an) ← C_1
- Lowest cost for a given $H(\Delta t)^{0.5}$: 1020 steel (an) ← C_2

SUMMARY

- Material costs fluctuate but rise over the long term as:

  --rich deposits are depleted,

  --energy costs increase.

- Recycled materials reduce energy use significantly.
- Materials are selected based on:

  --performance or cost indices.

- Examples:

  --design of minimum mass, maximum strength of:

  - shafts under torsion,
  - bars under tension,
  - plates under bending,

  --selection of materials to optimize more than one property:

  - material for a magnet coil.
  - analysis does not include cost of operating the magnet.